kittel

4.1. (a) $T = \frac{1}{2}Mv^2 = \frac{1}{2}M\left(\frac{du_s}{dt}\right)^2$ per atom.

In the context of only considering nearest neighbor interactions, we use the harmonic oscillator potential.

$$U = \frac{1}{2}kx^2 = \frac{1}{2}C(u_s - u_{s+1})^2 \text{ between } s \text{ and } s+1.$$

$$\Rightarrow \boxed{E = \frac{1}{2}M\sum_s\left(\frac{du_s}{dt}\right)^2 + \frac{1}{2}C\sum_s(u_s - u_{s+1})^2}$$

In the second sum, we don't have to sum both neighbors of an atom (i.e. $(u_s-u_{s+1})^2 + (u_s - u_{s-1})^2$) because the potential is stored in the "spring" not in the atom. In a linear lattice each atom has on average one spring connected to its neighbor on the right.

(b). $\langle \frac{1}{2}M\left(\frac{du_s}{dt}\right)^2\rangle = \frac{1}{2}Mu^2\langle\cos^2(\omega t - sKa)\rangle$

$$= \boxed{\frac{1}{4}Mu^2}$$

$$\langle\frac{1}{2}C(u_s-u_{s+1})^2\rangle = \frac{1}{2}C\langle u^2[\cos(\omega t - sKa) - \cos(\omega t-(s+1)Ka)]^2\rangle$$

$$= \frac{1}{2}Cu^2\langle\left[\cos^2(X) + \cos^2(X-Ka) - 2\cos(X)\cos(X-Ka)\right]\rangle$$

$$= \frac{1}{2}Cu^2\left[1 - 2\langle\cos X\cos(X-Ka)\rangle\right], \quad X = \omega t - sKa$$

$$= \frac{1}{2}Cu^2\left[1 - 2\left[\frac{1}{2}\langle\cos(2X-Ka)\rangle + \frac{1}{2}\langle\cos Ka\rangle\right]\right]$$

$$= \boxed{\frac{1}{2}Cu^2[1-\cos Ka]}$$

Davidson Cheng
12.23.2023